Definitionen

fnord

Konstanten

Bethe-Weizsäcker

$a_V = 15.67\,\mathrm{MeV}$, $a_S = 17.23\,\mathrm{MeV}$, $a_C = 0.714\,\mathrm{MeV}$, $a_A = 93.15\,\mathrm{MeV}$, $\delta = 11.2\,\mathrm{MeV}$

Spin & magn. Momente

$g_s = 2.002$ (e^-, μ^-), $g_p = 5.585$, $g_n = -3.825$

$\mu_K = e\hbar/(2m_p)$, $\mu_p = 2.7928\mu_K$, $\mu_n = -1.913\mu_K$

Schräge Einheiten

$1\,\mathrm{Ci} = 3.7 \times 10^{10}\,\mathrm{Hz}$

$1\,\mathrm{u} = 1.660\,538\,783\,16 \times 10^{-27}\,\mathrm{kg}$

$1\,\mathrm{b} = 1 \times 10^{-28}\,\mathrm{m}^2 = 100\,\mathrm{fm}^2$

Hilfreiche Werte

Massen

$m_{1\mathrm{H}} = m_p + m_e = 1.007\,825\,046\,\mathrm{u}$

$r_0 \approx 1.2\,\mathrm{fm}$

1. Mathematisches

Taylor-Reihe

$f(x) = \sum_{i=0}^n \frac{1}{i!}(x-x_0)^i \frac{\mathrm{d}^i f}{\mathrm{d}x^i}(x_0) + R_n$

Fourier-Reihe

$u(x) = \frac{a_0}{2} \sum_{n=1}^\infty (a_n \cos(nx) + b_n \sin(nx))$

$a_n := \frac{1}{\pi}\int_{-\pi}^{\pi} \mathrm{d}x u(x)\cos(nx)$,

$b_n := \frac{1}{\pi}\int_{-\pi}^{\pi} \mathrm{d}x u(x)\sin(nx)$

Fourier-Transformation:

$\mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{d}t e^{-i\omega t} f(t) \equiv \mathcal{F}_t(f)$

$\mathcal{F}^{-1}(f)(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{d}\omega e^{i\omega t} \tilde{f}(\omega)$

$\int_{-\infty}^{\infty} \mathrm{d}t e^{-i(\omega-\omega')t} = 2\pi\delta(\omega-\omega')$

Trigonometrische Fkt.

$\cos z = \frac{1}{2}\left(e^{iz} + e^{-iz}\right)$

$\sin z = \frac{1}{2i}\left(e^{iz} - e^{-iz}\right)$

$\sinh z = \frac{1}{2}\left(e^{z} - e^{-z}\right)$

$\cosh z = \frac{1}{2}\left(e^{z} + e^{-z}\right)$

Trigonometr. Add. Th.

$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$,

$\cos(x \pm y) = \cos x \cos y \mp \sin x \cos y$,

$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$, $\cot(x \pm y) = \frac{\cot x \cot y \pm 1}{\cot y \mp \cot x}$

Trigonometr. Th., sonstg.

$\sin(x+y)\sin(x-y) = \cos^2 y - \cos^2 x = \sin^2 x - \sin^2 y$,

$\cos(x+y)\cos(x-y) = \cos^2 y - \sin^2 x = \cos^2 y + \cos^2 x - 1 = 1 - \sin^2 x - \sin^2 y$,

$\sin x \pm \sin y = 2\sin\frac{x \pm y}{2}\cos\frac{x \mp y}{2}$,

$\cos x \pm \cos y = \pm 2\cos\frac{x \pm y}{2}\cos\frac{x \mp y}{2}$

Verkettete trigonom. Fkt.

$\sin(\arccos x) = \cos(\arcsin x) = \sqrt{1-x^2}$,

$\sin(\arctan x) = \frac{x}{\sqrt{1+x^2}}$,

$\cos(\arctan x) = \frac{1}{\sqrt{1+x^2}}$,

$\tan(\arcsin x) = \frac{x}{\sqrt{1-x^2}}$,

$\tan(\arccos x) = \frac{\sqrt{1-x^2}}{x}$

Integrale

$\int_{-\infty}^{\infty} \mathrm{d}x e^{-cx^2} = \sqrt{\frac{\pi}{c}}$

$\sqrt{2\pi} \int_{-\infty}^{\infty} \mathrm{d}x e^{-\frac{x}{a} + i(k0-k)x} = \sqrt{\frac{a}{2}}\, e^{-\frac{1}{2}(k_0-k)^2}$

Näherungen

$(1 \pm x)^n \approx 1 \pm nx$, $\sqrt{1 \pm x} \approx 1 \pm \frac{1}{2}x$, $\sin(x) \approx x$, $\cos(x) \approx 1 - \frac{x^2}{2}$,

$e^x \approx 1 + x$, $\ln(1+x) \approx x$,

Raumwinkelelement

$\Omega = \frac{A}{r^2}$, $\mathrm{d}\Omega = \sin(\vartheta)\mathrm{d}\vartheta\mathrm{d}\varphi$

2. Elektrodynamik

Coulomb-Potential

$V(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r}$

Photonen

$p = E/c = 2\pi\hbar/\lambda$, $m = \hbar k/c$,

$E = \hbar\omega$

Feldenergie

$E_F = \frac{\varepsilon_0}{2}\int \mathrm{d}^3 r \vec{E}^2(\vec{r})$

homogene Kugel

$$\vec{E}(\vec{r}) = \frac{Ze}{4\pi\varepsilon_0}\begin{cases} \frac{\vec{r}}{R^3} & |r| < R \\ \frac{\vec{r}}{r^3} & |r| > R \end{cases}$$

$E_F = \frac{3}{5}\frac{Z^2e^2}{4\pi\varepsilon_0}\frac{1}{R}$

3. Relativistik Grundlagen

Definitionen

$\beta = \frac{v}{c}$,

$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = (1-\beta^2)^{-1/2}$

$E^2 = m^2c^4 + p^2c^2$

Näherungen

Klassisch ($p \approx 0$)

$E \approx mc^2$

Ultrarelativistisch ($m \approx 0$)

$E \approx pc$

Energie-/Impulserhaltung

für 4 Partikel-Kollision

$m_Ac^2 + T_A + m_bc^2 + T_b = m_cc^2 + T_c + m_Dc^2 + T_D$

$p_b = p_c\cos\theta + p_D\cos\varphi$,

$0 = p_c\sin\theta - p_D\sin\varphi$

Doppler-Effekt

f: beobachtete Frequenz, f_0: Ruhefrequenz, v: Beobachtergeschw.

$f = f_0\sqrt{\frac{c+v}{c-v}}$

Doppler-Intensität

$I(\nu) = I_0 e^{-\frac{1}{2}\frac{(\nu-\nu_0)^2}{\Delta\nu^2}}$, $\Delta\nu = \frac{\nu_0}{c}\sqrt{\frac{k_BT}{m}}$

Halbwertsbreite: $2\sqrt{2\ln 2}\,\Delta\nu$

4. Kernphysik

Massenspektrometer

$\frac{m}{q} = \frac{rB^2}{E}$

Penning-Falle

$\omega_c = \frac{qB}{m}$

$d^2 = \frac{1}{2}(z_0^2 + r_0^2/2)$

$\omega_z = \sqrt{\frac{qU}{md^2}}$

$\omega_+ = \frac{\omega_c}{2} + \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}} \approx \omega_c - \frac{U}{2d^2B}$

$\omega_- = \frac{\omega_c}{2} - \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}} \approx \frac{U}{2d^2B}$

Für gebundene Bewegung muss gelten: $\omega_c^2 - 2\omega_z^2 > 0$

Bethe-Weizsäcker

$E_B(Z,A) = -a_VA + a_SA^{2/3} + a_C\frac{Z^2}{A^{1/3}} + a_a\frac{(N-Z)^2}{4A} + B_P$

$$B_P = \begin{cases} +\delta & \text{gg} \\ 0 & \text{ug/gu} \\ -\delta & \text{uu} \end{cases}$$

$m(Z,A) = Zm_{1\mathrm{H}} + (A-Z)m_n + E_B(Z,A)/c^2$

Separationsenergie

$S_n = B(Z,A) - B(Z,A-1)$,

$S_p = B(Z,A) - B(Z-1,A-1)$

Röntgen-Isotopen-Shift

$E_K(A) - E_K(A') = E_{1s}(A) - E_{1s}(A') = -\frac{2}{5}\frac{Z^4e^2}{4\pi\varepsilon_0}\frac{r_0^2}{a_0^3}\left(A^{2/3} - A'^{2/3}\right)$

Spiegelkerne

$\Delta E_C = \frac{3e^2}{20\pi\varepsilon_0 R}(2Z-1)$

Luminosität

Teilchenfluss $\phi_a = \frac{\dot{N}_a}{A} = n_av_a$

Reaktionsrate $\dot{N} = \phi_aN_b\sigma_b$

$L = \phi_aN_b = n_av_a \Rightarrow \dot{N} = L\sigma$

$\mathcal{L} = \int \mathrm{d}t\, L$

Streuung

Rutherford (Klassisch)

$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Ruth}} = \left(\frac{zZe^2}{16\pi\varepsilon_0E}\right)^2\frac{1}{\sin^4(\theta/2)}$

Mott (Relativistik)

$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}} = \left(1 - \frac{v^2}{c^2}\sin^2(\theta/2)\right)\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Ruth}}$

Formfaktor

$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) = |F(\vec{q})|^2\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}}$

$F(\vec{q}) = \int \mathrm{d}^3r\, e^{i\vec{q}\vec{r}/\hbar}\rho(\vec{r})$

Schwere Kerne

$\rho(r) = \frac{\rho_0}{1+e^{(r-c)/a}}$

$a \approx 0.54\,\mathrm{fm}$, $c \approx A^{1/3} \cdot 1.07\,\mathrm{fm}$,

Kerndrehimpulse

gg-Kerne

$I = 0$ (Ausgleich)

gu/ug-Kerne

$I = j_{\text{ungerade}}$

uu: Nordheimregeln

$N = j_p - l_p + j_n + l_n$: Nordheimzahl

Schwache Nordheimregel: ↑↑ + ↑↑

$N = 0$. $j_{ges;max} = j_1 + j_2$

Starke Nordheimregel: ↑↓ + ↑↑

$N = \pm 1$. $j_{ges;max} = |j_1 - j_2|$

↑↓⇔↓↑, ↑↑⇔↓↓

Kerndrehimpulse Bsp.

Analog zur Aufgabe.

p: $\overbrace{(1s_{1/2})^2}^{s=0}\overbrace{(1p_{3/2})^4}^{s=0}\overbrace{(1p_{1/2})^2}^{s=0}\overbrace{(1d_{5/2})^1}^{s\neq 0}$

n: $\underbrace{(1s_{1/2})^2}_{s=0}\underbrace{(1p_{3/2})^{4-3}}_{s\neq 0}\underbrace{(1p_{1/2})^2}_{s=0}\underbrace{(1d_{5/2})^1}_{s\neq 0}$

Daraus folgen Drehimpulse:

I	$p: 1d_{5/2}$	$\Rightarrow l = 2, s = 1/2$
		$j = 5/2 = l + s$
II	$n: 1d_{5/2}$	$\Rightarrow l = 2, s = 1/2$
		$j = 5/2 = l + s$
III	$n: 1p_{3/2}$	$\Rightarrow l = 1, s = 1/2$
		$j = 3/2 = l + s$

I+II: Schwache Nordheim-Regel: ↑↑ + ↑↑

$j_{ges;max} = j_I + j_{II} = 5 \Rightarrow j_{ges} \in \{5,4,3,2,1,0\}$

(I+II)+III: analog (wählen $j_{I+II} = 2$)

$j_{ges;max} = j_{I+II} + j_{III} = 7/2 \Rightarrow j_{ges} \in \{\frac{7}{2}, \frac{5}{2}, \frac{3}{2}, \frac{1}{2}\}$.

Parität: $P = (-1)^l$, multiplikativ

$P = P_IP_{II}P_{III} = 1 \cdot 1 \cdot (-1) = -1$

5. Kernreaktionen

Nomenklatur

A: Target; b: Projektil; c: Ejektil; D: Ergebnis

Labor vs. Schwerpunkt

Laborsystem

$m_{\{A,b,c,D\}}$, $v_{\{A,b,c,D\}}$, $T_{\{A,b,c,D\}}$.

Schwerpunktsystem

$u_{\{A,b,c,D\}}$, $T^M_{\{A,b,c,D\}}$.

Trafo

$T^M_X = \frac{m_A}{m_A+m_b}T_X$

Q-Wert

$T_c^{1/2} = \frac{\sqrt{m_bm_cT_b}\cos\theta \pm \sqrt{\alpha}}{m_c+m_D}$,

$\alpha := m_bm_cT_b\cos^2(\theta) + (m_D + m_c)(m_DQ + (m_D - m_b)T_b)$

$Q = T_c\left(1 + \frac{m_c}{m_D}\right) - T_b\left(1 - \frac{m_b}{m_D}\right) - 2\sqrt{\frac{m_bm_c}{m_D^2}T_cT_b}\cos(\theta)$

Reaktionsbedingung

$T_b > -Q\frac{m_D+m_c}{m_D+m_c+m_b}$

6. Zerfall

Zerfallsgesetz

Zerfallskonst. $\lambda = -\frac{\partial_t N}{N}$

$N(t) = N_0e^{-\lambda t}$, $T_{1/2} = \frac{\ln 2}{\lambda}$

mittl. Lebensd. $\tau = \lambda^{-1}$

Messung $\Delta t \ll \tau$, $\Delta t \ll T_{1/2}$

$\Delta N/\Delta t = \lambda N_0e^{-\lambda t}$

Aktivität

$A(t) = \lambda N(t) = A_0e^{-\lambda t}$

α-Zerfall

$E_\alpha = (m(Z,A) - m(Z-2,A-4) - m_\alpha)c^2$

$T_\alpha = \frac{Q}{1+m_\alpha/m_{X'}}$

$\ln T_{1/2} \propto 1/\sqrt{E_\alpha}$

Wahrscheinlichkeit

Transmission $T \approx e^{-G}$, $G := \frac{2\sqrt{2m}}{\hbar}\int_R^{R'}\mathrm{d}r\sqrt{Z_1Z_2e^2/r - E}$

$T_{1/2} \propto \frac{1}{\lambda} \propto \frac{1}{T} \propto e^G$

Spin=0 $\Rightarrow j = l \Rightarrow P = (-1)^{l\alpha}$

β-Zerfall

β^--Zerfall: $n \to p + e^- + \overline{\nu}$

$Q_{\beta-} \approx (m({}^A_Z\mathrm{X}) - m({}_{Z+1}^{\;\;A}\mathrm{X}'))c^2 = T_e + E_{\overline{\nu}}$

β^+-Zerfall: $p \to n + e^+ + \nu$

$Q_{\beta+} \approx (m({}^A_Z\mathrm{X}) - m({}_{Z+1}^{\;\;A}\mathrm{X}') - 2m_e)c^2$

EC: $e^- + {}^A\mathrm{X} \to {}_{Z-1}^{\;\;A}(X') + \nu$

$Q_\varepsilon \approx (m({}^A_Z\mathrm{X}) - m({}_{Z-1}^{\;\;A}\mathrm{X}'))c^2 - B_n$, B_n: Bindungsenergie des eingefangenen e

Kernspaltung

Verformung in Rotationsellipsoid: $a = R(1+\varepsilon)$, $b = R/\sqrt{1-\varepsilon}$

$\Delta E^{\text{Surf}} = \frac{2\varepsilon^2}{5}a_sA^{2/3}\varepsilon^2$

$\Delta E^{\text{Coul}} = -\frac{\varepsilon^2}{a}C\frac{Z(Z-1)}{A^{1/3}}$

$X_S := \frac{\Delta E^{\mathrm{C}}}{\Delta E^{\mathrm{S}}} = \frac{a_CZ^2}{2a_SA}$ Kern stabil wenn $X_S > 1$

Zerfallsketten

$N_A \to N_B \to N_C$, λ_A, λ_B, λ_C

$N_B(t) = -\frac{\lambda_A}{\lambda_A - \lambda_B} N_{A;0} e^{-\lambda_A t} + N_{B;0} e^{-\lambda_B t} + \frac{\lambda_A}{\lambda_A - \lambda_B} N_{A;0} e^{-\lambda_B t}$

7. Anhang

Kernniveaus